Mathematics Department , Faculty of Science ,Tanta University		
Branch: Math. Dept.		Sub-branch : Mathematics
Examination for : Level three		Term: first Term 2016-2017
Course Title: Mathematical Logic and Boolean Algebra		Course Code: MA3113
Date: 9/1/2017	Total Mark: 100	Time Allowed: 2 Hours

Answer the following questions:

First: Mathematical Logic.

Question 1 (30 marks):

a) Show that $(\neg(A \to (B \vee C)) \to (A \wedge (\neg B \wedge \neg C))$ is a wff. **(10 marks)**

b) Is $(((P \to Q) \to P) \to P)$ a tautology?

Define σ_k recursively as follows: $\sigma_0 = P \to Q$ and $\sigma_{k+1} = (\sigma_k \to P)$. For which values of k is σ_k is a tautology? **(10 marks)**

c) Let G be the following three-place Boolean function:

$$G(F,F,F) = F, \quad G(T,F,F) = T, \quad G(F,F,T) = T, \quad G(T,F,T) = F,$$
$$G(F,T,F) = T, \quad G(T,T,F) = F, \quad G(F,T,T) = F, \quad G(T,T,T) = F.$$

Find a wff, using at most the connectives $\vee$, $\wedge$, and $\neg$ that realizes G. **(10 marks)**

Question 2 (20 marks):

a) In the first-order logic language, define the following:

the terms, an atomic formula, the well -formed formula. **(9 marks)**

b) Rewrite the following wff in a way which explicitly lists each symbol in actual order. Say which variable occur free in the wff:

$$\forall v_1 A v_1 \wedge B v_1 \to \exists v_2 \neg C v_2 \vee D v_2.$$ **(6 marks)**

c) In the language of elementary number theory, translate the following sentence in a more formal way:

" Any nonzero natural number is the successor some number". **(5 marks)**

Second: Boolean Algebra.

Question 3 (20 marks):

a) Let $f(x, y, z)$ be the Boolean function represented by Table (1), then:

(i) Find $f(x, y, z)$. **(2 marks)**

(ii) Represent $f(x, y, z)$ by logic and series-parallel circuits and then find the differences between the two circuit types. **(5 marks)**

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TANTA UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

Final Term Exam for 3rd year students of Mathematics
First Term 2016-2017

	Course Title: Algebra (1)	Course Code: MA3107
Date: 4/1/2017	Total Mark: 150 Marks	Time Allowed: 2 Hours

Answer the following questions:

Question 1 (30 marks):

Prove that the set, $I(G)$, of all inner automorphisms forms a group under composition of maps. Show that $I(G) \cong G/Z$, then write the elements of the group $I(D_4)$.

Question 2 (30 marks):

a- Find the derived group of the alternating group A_4. **(15 marks)**

b- Let $\varphi: G \to H$ be a homomorphism and let S be a subgroup of H. Prove that $\varphi^{-1}(S) = \{x \in G: \varphi(x) \in S\}$ is a subgroup of G containing $ker\varphi$. **(15 marks)**

Question 3 (45 marks):

a- Consider the symmetric group S_5. Let $\rho = (1)(23)(45)$. Write all the elements of S_5 that are conjugate to ρ, hence determine all the elements of the centralizer subgroup $C(\rho)$. **(20 marks)**

b- Without doing any calculations in $Aut(Z_{20})$, prove that $Aut(Z_{20})$ is not cyclic. **(10 marks)**

c- Find the complete list of the distinct isomorphism classes of abelain groups of order 100. **(15 marks)**

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<u>Question 4:</u> (Maximum Mark: 15)

a. Consider the following ordinary differential equation with $x \in \mathbb{R}^+$:

$$xy'' + 2y' + xy = 0.$$

i. Show that $y(x) = \frac{\cos x}{x}$ is a solution of this equation.

ii. Find the second independent solution.

iii. Write out an expression for the general solution.

<u>Question 5:</u> (Maximum Mark: 40)

Solve the following ordinary differential equations:

i. $y'' - 2y' - 48y = 5e^{-6x} + (x-2)e^{-8x}$.

ii. $x^2y'' + 5xy' + 4y = \frac{\sec^{-1} \ln x}{x^2}$

iii. $\dot{x} - 2x(t) - 8y(t) = f(t),\ \dot{x} + 2\dot{y} - 4y = t,\ x(0) = 0, y(0) = 0,$

$$f(t) = \begin{cases} t, & 0 \le t < 10 \\ 10, & t \ge 10. \end{cases}$$

The End of Exam

With Best Wishes

Third question: **(36 Marks)**

(a) If $\int_a^b f\,dg$ exists. Prove that

(1) $\int_a^b g\,df$ exists (2) $\int_a^b g\,df = g(b)f(b) - g(a)f(a) - \int_a^b f\,dg$

(b) Let f be any function with continuous derivative on $[a,b]$.

Let $a = a_0 < a_1 < \cdots < a_N = b$ and define

$$g(x) = \begin{cases} C_i ; & a_{i-1} < x \le a_i \\ C_0 ; & x = a \end{cases} ; \quad i = \overline{1,N}$$

where $C_0, C_1, \cdots, C_N$ are constants. Prove that

(1) $\int_a^b g(x)\,df(x)$ exists (2) $\int_a^b g(x) f'(x)\,dx = C_N f(b) - C_0 f(a) - \sum_{j=0}^{N-1} f(a_j).(C_{j+1} - C_j)$

(c) Show that the function $f(x) = \begin{cases} x \sin\frac{1}{x} ; & x \ne 0 \\ 0 ; & x = 0 \end{cases}$ is unbounded variation on $[0,1]$.

Fourth Question **(38 Marks)**

(a) Find the complex form of the Fourier series for the complex function $f(x)$. Hence if $\{C_n\}_{-\infty}^{\infty}$ have a bounded variation and $\lim_{n\to\infty} C_n = \lim_{n\to\infty} C_{-n} = 0$.

Prove that $\sum_{n=-\infty}^{\infty} C_n e^{inx}$ converges uniformly on every interval $[c,d]$, where either $-\pi \le c < d < 0$ or $0 < c < d \le \pi$.

(b) Let f and g be two continuous functions on the interval $[-\pi,\pi]$ which have the same Fourier coefficients. Prove that $f(x) = g(x)$ for all $x \in [-\pi,\pi]$.

(c) Suppose that $f(x)$ and $g(x)$ bounded variation on $[a,b]$. Prove that $f-g$ and fg are of bounded variation.

(Best wishes)

Examiners:	1- Prof. Dr. S. Abdel Aziz 2- Dr. Usama A. Embaby